

4.1 ΜΕΤΑΣΧΗΜΑΤΙΣΜΟΙ ΤΡΙΓΩΝΟΜΕΤΡΙΚΩΝ ΠΑΡΑΣΤΑΣΕΩΝ

Δραστηριότητες σελ. 185 (Μετασχηματισμοί τριγωνομετρικών παραστάσεων)

<p>1.</p>	<p>(α) Έχουμε</p> $2\eta\mu(4\omega)\eta\mu\omega = \sigma\upsilon\nu(4\omega - \omega) - \sigma\upsilon\nu(4\omega + \omega)$ $= \sigma\upsilon\nu(3\omega) - \sigma\upsilon\nu(5\omega)$ <p>(β) Έχουμε</p> $\sigma\upsilon\nu(8\psi)\sigma\upsilon\nu(2\psi) = \frac{1}{2}(\sigma\upsilon\nu(8\psi - 2\psi) + \sigma\upsilon\nu(8\psi + 2\psi))$ $= \frac{1}{2}(\sigma\upsilon\nu(6\psi) + \sigma\upsilon\nu(10\psi))$ <p>(γ) Έχουμε</p> $\sigma\upsilon\nu(2x)\eta\mu x = \frac{1}{2}(\eta\mu(2x + x) + \eta\mu(x - 2x))$ $= \frac{1}{2}(\eta\mu(3x) + \eta\mu(-x)) = \frac{1}{2}(\eta\mu(3x) - \eta\mu x)$
<p>2.</p>	<p>(α) $2\eta\mu 135^\circ \sigma\upsilon\nu 15^\circ = \eta\mu(135^\circ + 15^\circ) + \eta\mu(135^\circ - 15^\circ)$</p> $= \eta\mu(150^\circ) + \eta\mu(120^\circ)$ $= \eta\mu(30^\circ) + \eta\mu(60^\circ)$ $= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$ <p>(β) $\eta\mu\left(\frac{11\pi}{12}\right)\eta\mu\left(\frac{7\pi}{12}\right) = \frac{1}{2}\left[\sigma\upsilon\nu\left(\frac{11\pi}{12} - \frac{7\pi}{12}\right) - \sigma\upsilon\nu\left(\frac{11\pi}{12} + \frac{7\pi}{12}\right)\right]$</p> $= \frac{1}{2}\left[\sigma\upsilon\nu\left(\frac{4\pi}{12}\right) - \sigma\upsilon\nu\left(\frac{18\pi}{12}\right)\right]$ $= \frac{1}{2}\left[\sigma\upsilon\nu\left(\frac{\pi}{3}\right) - \sigma\upsilon\nu\left(\pi + \frac{\pi}{2}\right)\right]$ $= \frac{1}{2}\left[\frac{1}{2} + \eta\mu(\pi)\right] = \frac{1}{4}$
<p>3.</p>	<p>(α) Έχουμε</p> $\sigma\upsilon\nu(4\omega) + \sigma\upsilon\nu(2\omega) = 2\sigma\upsilon\nu\left(\frac{4\omega - 2\omega}{2}\right) \cdot \sigma\upsilon\nu\left(\frac{4\omega + 2\omega}{2}\right)$ $= 2\sigma\upsilon\nu\left(\frac{2\omega}{2}\right) \cdot \sigma\upsilon\nu\left(\frac{6\omega}{2}\right) = 2\sigma\upsilon\nu(\omega) \cdot \sigma\upsilon\nu(3\omega)$ <p>(β) Έχουμε</p> $\eta\mu(3\alpha) + \eta\mu(2\alpha) = 2\eta\mu\left(\frac{3\alpha + 2\alpha}{2}\right) \cdot \sigma\upsilon\nu\left(\frac{3\alpha - 2\alpha}{2}\right)$ $= 2\eta\mu\left(\frac{5\alpha}{2}\right) \cdot \sigma\upsilon\nu\left(\frac{\alpha}{2}\right)$ <p>(γ) Έχουμε</p> $\eta\mu(7x) + \sigma\upsilon\nu(2x) = \eta\mu(7x) + \eta\mu\left(\frac{\pi}{2} - 2x\right)$ $= 2\eta\mu\left(\frac{7x + \frac{\pi}{2} - 2x}{2}\right) \cdot \sigma\upsilon\nu\left(\frac{7x - \left(\frac{\pi}{2} - 2x\right)}{2}\right)$ $= 2\eta\mu\left(\frac{10x + \pi}{4}\right) \cdot \sigma\upsilon\nu\left(\frac{18x - \pi}{4}\right)$

4.

(α) Έχουμε

$$\frac{2\eta\mu(3x)\sigma\upsilon\nu x - \eta\mu(2x)}{2\eta\mu(3x)\eta\mu x + \sigma\upsilon\nu(4x)} = \frac{\eta\mu(4x) + \eta\mu(2x) - \eta\mu(2x)}{\sigma\upsilon\nu(2x) - \sigma\upsilon\nu(4x) + \sigma\upsilon\nu(4x)}$$

$$= \frac{\eta\mu(4x)}{\sigma\upsilon\nu(2x)} = \frac{2\eta\mu(2x)\sigma\upsilon\nu(2x)}{\sigma\upsilon\nu(2x)} = 2\eta\mu(2x)$$

(β) Έχουμε

$$2\eta\mu\left(\frac{\pi}{4} + x\right)\eta\mu\left(\frac{\pi}{4} - x\right)$$

$$= \sigma\upsilon\nu\left(\frac{\pi}{4} + x - \left(\frac{\pi}{4} - x\right)\right) - \sigma\upsilon\nu\left(\frac{\pi}{4} + x + \left(\frac{\pi}{4} - x\right)\right)$$

$$= \sigma\upsilon\nu(2x) - \sigma\upsilon\nu\left(\frac{\pi}{2}\right) = \sigma\upsilon\nu(2x)$$

(γ) Έχουμε

$$\frac{\eta\mu\omega + \eta\mu\varphi}{\sigma\upsilon\nu\omega + \sigma\upsilon\nu\varphi} = \frac{2\eta\mu\left(\frac{\omega + \varphi}{2}\right)\sigma\upsilon\nu\left(\frac{\omega - \varphi}{2}\right)}{2\sigma\upsilon\nu\left(\frac{\omega + \varphi}{2}\right)\sigma\upsilon\nu\left(\frac{\omega - \varphi}{2}\right)} = \frac{\eta\mu\left(\frac{\omega + \varphi}{2}\right)}{\sigma\upsilon\nu\left(\frac{\omega + \varphi}{2}\right)} = \varepsilon\varphi\left(\frac{\omega + \varphi}{2}\right)$$

(δ) Έχουμε

$$\frac{\sigma\upsilon\nu(2\alpha) + \sigma\upsilon\nu(4\alpha)}{\sigma\upsilon\nu(2\alpha) - \sigma\upsilon\nu(4\alpha)} = \frac{2\sigma\upsilon\nu\left(\frac{2\alpha + 4\alpha}{2}\right)\sigma\upsilon\nu\left(\frac{2\alpha - 4\alpha}{2}\right)}{2\eta\mu\left(\frac{2\alpha + 4\alpha}{2}\right)\eta\mu\left(\frac{4\alpha - 2\alpha}{2}\right)}$$

$$= \frac{2\sigma\upsilon\nu\left(\frac{6\alpha}{2}\right)\sigma\upsilon\nu\left(-\frac{2\alpha}{2}\right)}{2\eta\mu\left(\frac{6\alpha}{2}\right)\eta\mu\left(\frac{2\alpha}{2}\right)} = \frac{\sigma\upsilon\nu(3\alpha)\overbrace{\sigma\upsilon\nu(-\alpha)}^{\sigma\upsilon\nu\alpha}}{\eta\mu(3\alpha)\eta\mu(\alpha)}$$

$$= \frac{\sigma\upsilon\nu(3\alpha)}{\eta\mu(3\alpha)} \cdot \frac{\sigma\upsilon\nu\alpha}{\eta\mu\alpha} = \sigma\varphi(3\alpha) \cdot \sigma\varphi\alpha$$

(ε) Έχουμε

$$\frac{\sigma\upsilon\nu\alpha - \sigma\upsilon\nu(5\alpha)}{\eta\mu\alpha + \eta\mu(5\alpha) + 2\eta\mu(3\alpha)}$$

$$= \frac{2\eta\mu\left(\frac{\alpha + 5\alpha}{2}\right)\eta\mu\left(\frac{5\alpha - \alpha}{2}\right)}{2\eta\mu\left(\frac{\alpha + 5\alpha}{2}\right)\sigma\upsilon\nu\left(\frac{\alpha - 5\alpha}{2}\right) + 2\eta\mu(3\alpha)}$$

$$= \frac{2\eta\mu\left(\frac{6\alpha}{2}\right)\eta\mu\left(\frac{4\alpha}{2}\right)}{2\eta\mu\left(\frac{6\alpha}{2}\right)\sigma\upsilon\nu\left(\frac{-4\alpha}{2}\right) + 2\eta\mu(3\alpha)}$$

$$= \frac{2\eta\mu(3\alpha)\eta\mu(2\alpha)}{2\eta\mu(3\alpha)\sigma\upsilon\nu(-2\alpha) + 2\eta\mu(3\alpha)}$$

$$= \frac{2\eta\mu(3\alpha)\eta\mu(2\alpha)}{2\eta\mu(3\alpha)\sigma\upsilon\nu(2\alpha) + 2\eta\mu(3\alpha)} = \frac{2\eta\mu(3\alpha)\eta\mu(2\alpha)}{2\eta\mu(3\alpha)(\sigma\upsilon\nu(2\alpha) + 1)}$$

$$= \frac{\eta\mu(2\alpha)}{\sigma\upsilon\nu(2\alpha) + 1} = \frac{2\eta\mu\alpha\sigma\upsilon\nu\alpha}{2\sigma\upsilon\nu^2\alpha} = \frac{\eta\mu\alpha}{\sigma\upsilon\nu\alpha} = \varepsilon\varphi\alpha$$

5.	$\eta\mu A = \sigma\upsilon\nu B + \sigma\upsilon\nu\Gamma \Leftrightarrow \eta\mu A = 2\sigma\upsilon\nu\left(\frac{B+\Gamma}{2}\right)\sigma\upsilon\nu\left(\frac{B-\Gamma}{2}\right)$ <p>(αφού $A + B + \Gamma = \pi$)</p> $\Leftrightarrow \eta\mu A = 2\sigma\upsilon\nu\left(\frac{\pi}{2} - \frac{A}{2}\right)\sigma\upsilon\nu\left(\frac{B-\Gamma}{2}\right) \Leftrightarrow 2\eta\mu\left(\frac{A}{2}\right)\sigma\upsilon\nu\left(\frac{A}{2}\right)$ $= 2\eta\mu\left(\frac{A}{2}\right)\sigma\upsilon\nu\left(\frac{B-\Gamma}{2}\right) \Leftrightarrow \sigma\upsilon\nu\left(\frac{A}{2}\right) = \sigma\upsilon\nu\left(\frac{B-\Gamma}{2}\right)$ <p>και αφού οι γωνίες $\frac{A}{2}$ και $\frac{B-\Gamma}{2}$ είναι οξείες, έχουμε ότι</p> $\frac{A}{2} = \frac{B-\Gamma}{2}, \quad \text{δηλ. } \frac{A}{\pi-(B+\Gamma)} = B-\Gamma, \quad \text{δηλ. } 2B = \pi, \quad \text{δηλ. } B = \frac{\pi}{2}$
6.	$\frac{\alpha - \beta}{\gamma} \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right) = \frac{2R\eta\mu A - 2R\eta\mu B}{2R\eta\mu\Gamma} \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)$ $= \frac{\eta\mu A - \eta\mu B}{\eta\mu\Gamma} \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)$ <p>(αφού $A + B + \Gamma = \pi$)</p> $= \frac{2\eta\mu\left(\frac{A-B}{2}\right)\sigma\upsilon\nu\left(\frac{A+B}{2}\right)}{\eta\mu\Gamma} \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)$ $= \frac{2\eta\mu\left(\frac{A-B}{2}\right)\sigma\upsilon\nu\left(\frac{\pi}{2} - \frac{\Gamma}{2}\right)}{\eta\mu\Gamma} \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)$ $= \frac{2\eta\mu\left(\frac{A-B}{2}\right)\eta\mu\left(\frac{\Gamma}{2}\right)}{2\eta\mu\left(\frac{\Gamma}{2}\right)\sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)} \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right) = \eta\mu\left(\frac{A-B}{2}\right)$
7.	<p>(α) Έχουμε</p> $\eta\mu A - \eta\mu B + \eta\mu\Gamma = 2\eta\mu\left(\frac{A-B}{2}\right)\sigma\upsilon\nu\left(\frac{A+B}{2}\right) + 2\eta\mu\left(\frac{\Gamma}{2}\right)\sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)$ $= 2\eta\mu\left(\frac{A-B}{2}\right)\sigma\upsilon\nu\left(\frac{\pi}{2} - \frac{\Gamma}{2}\right) + 2\eta\mu\left(\frac{\Gamma}{2}\right)\sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)$ $= 2\eta\mu\left(\frac{A-B}{2}\right)\eta\mu\left(\frac{\Gamma}{2}\right) + 2\eta\mu\left(\frac{\Gamma}{2}\right)\sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)$ $= 2\eta\mu\left(\frac{\Gamma}{2}\right)\left[\eta\mu\left(\frac{A-B}{2}\right) + \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)\right]$ $= 2\eta\mu\left(\frac{\Gamma}{2}\right)\left[\eta\mu\left(\frac{A-B}{2}\right) + \sigma\upsilon\nu\left(\frac{\pi}{2} - \frac{A+B}{2}\right)\right]$ $= 2\eta\mu\left(\frac{\Gamma}{2}\right)\left[\eta\mu\left(\frac{A-B}{2}\right) + \sigma\upsilon\nu\left(\frac{\Gamma}{2}\right)\right]$ $= 2\eta\mu\left(\frac{\Gamma}{2}\right)\left[\eta\mu\left(\frac{A-B}{2}\right) + \eta\mu\left(\frac{A+B}{2}\right)\right]$ $= 2\eta\mu\left(\frac{\Gamma}{2}\right)\left[2\eta\mu\left(\frac{\frac{A-B}{2} + \frac{A+B}{2}}{2}\right)\sigma\upsilon\nu\left(\frac{\frac{A-B}{2} + \frac{A+B}{2}}{2}\right)\right]$ $= 4\eta\mu\left(\frac{\Gamma}{2}\right)\eta\mu\left(\frac{A}{2}\right)\sigma\upsilon\nu\left(\frac{B}{2}\right)$

(β) Έχουμε

$$\begin{aligned} & \sigma\nu\nu A + \sigma\nu\nu B + \sigma\nu\nu\Gamma \\ &= 2\sigma\nu\nu\left(\frac{A+B}{2}\right)\sigma\nu\nu\left(\frac{A-B}{2}\right) + 1 - 2\eta\mu^2\left(\frac{\Gamma}{2}\right) \\ &= 2\sigma\nu\nu\left(\frac{\pi}{2} - \frac{\Gamma}{2}\right)\sigma\nu\nu\left(\frac{A-B}{2}\right) + 1 - 2\eta\mu^2\left(\frac{\Gamma}{2}\right) \\ &= 2\eta\mu\left(\frac{\Gamma}{2}\right)\sigma\nu\nu\left(\frac{A-B}{2}\right) + 1 - 2\eta\mu^2\left(\frac{\Gamma}{2}\right) \\ &= 1 + 2\eta\mu\left(\frac{\Gamma}{2}\right)\left[\sigma\nu\nu\left(\frac{A-B}{2}\right) - \eta\mu\left(\frac{\Gamma}{2}\right)\right] \\ &= 1 + 2\eta\mu\left(\frac{\Gamma}{2}\right)\left[\sigma\nu\nu\left(\frac{A-B}{2}\right) - \sigma\nu\nu\left(\frac{A+B}{2}\right)\right] \\ &= 1 + 2\eta\mu\left(\frac{\Gamma}{2}\right) \cdot \left[2\eta\mu\left(\frac{\frac{A+B}{2} + \frac{A-B}{2}}{2}\right)\eta\mu\left(\frac{\frac{A+B}{2} - \frac{A-B}{2}}{2}\right)\right] \\ &= 1 + 2\eta\mu\left(\frac{\Gamma}{2}\right) \cdot \left[2\eta\mu\left(\frac{A}{2}\right)\eta\mu\left(\frac{B}{2}\right)\right] \\ &= 1 + 4\eta\mu\left(\frac{A}{2}\right)\eta\mu\left(\frac{B}{2}\right)\eta\mu\left(\frac{\Gamma}{2}\right) \end{aligned}$$