Δραστηριότητες σελ. 25 (Ενότητα 3.5: Κανόνες ολοκλήρωσης)

1.

(α) $\int x^4 dx = \frac{x^5}{5} + c$

(β) $\int x^{-2} dx = -\frac{1}{x} + c$

(γ) $\int x^{\frac{3}{4}} dx = \frac{4x^{\frac{7}{4}}}{7} + c$

(δ) $\int \frac{1}{u^4} du = -\frac{1}{3u^3} + c$

(ε) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$

(στ) $\int \frac{2}{\sqrt[3]{u}} du = 3 \cdot u^{\frac{2}{3}} + c$

(ζ) $\int (u^2 - 3u + 1) du = \int u^2 du - 3\int u du + \int du = \frac{u^3}{3} - 3 \cdot \frac{u^2}{2} + u + c$

(η) $\int (x + 2\sqrt{x} - \pi) dx = \int x dx + 2\int \sqrt{x} dx - \pi \int dx = \frac{x^2}{2} + 2 \cdot \frac{2}{3} x^{\frac{3}{2}} - \pi x + c$

$= \frac{x^2}{2} + 4\frac{x^{\frac{3}{2}}}{3} - \pi x + c$

(θ) $\int \left(e^x + ex + \frac{e}{x}\right) dx = \int e^x dx + e\int x dx + e\int \frac{1}{x} dx = e^x + e\frac{x^2}{2} + \ln|x| + c$

(ι) $\int (u-5)^2 du = \int (u^2 - 10u + 25) du = \int u^2 du - 10\int u du + 25\int du = \frac{u^3}{3} - 10\frac{u^2}{2} + 25u + c$

$= \frac{u^3}{3} - 5u^2 + 25u + c$

(ια) $\int 2u(u^2 - 3) du = 2\int (u^3 - 3u) du = 2\left(\int u^3 du - 3\int u du\right) = 2\frac{u^4}{4} - 3u^2 + c = \frac{u^4}{2} - 3u^2 + c$

(ιβ) $\int \sqrt{x}(x-2) dx = \int \left(x^{\frac{3}{2}} - 2\sqrt{x}\right) dx = \int x^{\frac{3}{2}} dx - 2\int \sqrt{x} dx = \frac{2}{5}x^{\frac{5}{2}} - 2\frac{2}{3}x^{\frac{3}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + c$

(ιγ) $\int \left(\frac{u^4 - 5u^3 + 3u}{u^2}\right) du = \int \left(\frac{u^4}{u^2} - 5\frac{u^3}{u^2} + \frac{3}{u}\right) du = \int \left(u^2 - 5u + \frac{3}{u}\right) du$

$= \int u^3 du - 5\int u du + 3\int \frac{du}{u} = \frac{u^3}{3} - 5\frac{u^2}{2} + 3\ln|u| + c$

(ιδ) $\int \left(\frac{3x - x^2 + 2x^3}{x^3}\right) dx = \int \left(\frac{3x}{x^3} - \frac{x^2}{x^3} + 2\frac{x^3}{x^3}\right) dx = \int \left(\frac{3}{x^2} - \frac{1}{x} + 2\right) dx$

$= 3\int \frac{1}{x^2} dx - \int \frac{1}{x} dx + 2\int dx = -\frac{3}{x} - \ln|x| + 2x + c$

(ιε) $\int (\eta\mu\theta - \sigma\upsilon\nu\theta) d\theta = \int \eta\mu\theta d\theta - \int \sigma\upsilon\nu\theta d\theta = -\sigma\upsilon\nu\theta - \eta\mu\theta + c$

(ιστ) $\int (3\theta - \tau\varepsilon\mu^2\theta) d\theta = 3\int \theta d\theta - \int \tau\varepsilon\mu^2\theta d\theta = 3\frac{\theta^2}{2} - \varepsilon\varphi\theta + c$

(ιζ) $\int \left(\frac{1}{1+x^2} - 2x\right) dx = \int \frac{dx}{1+x^2} - \int (2x) dx = τοξεφx - x^2 + c$

(ιη) $\int \frac{2}{\sqrt{1-x^2}} dx = 2 \int \frac{dx}{\sqrt{1-x^2}} = 2τοξημx + c$

2. (Αφού για κάθε τιμή των παραμέτρων $κ$ και $λ$ οι συναρτήσεις $x \mapsto κx^{κ-2}$ και $x \mapsto 3x^5 + c$ είναι συνεχείς) έχουμε ($κ \neq 1$):

$$\int λx^{κ-2}\, dx = 3x^5 + c \Leftrightarrow λx^{κ-2} = (3x^5 + c)' \Leftrightarrow λx^{κ-2} = 15x^4$$

$$\Leftrightarrow \begin{cases} λ = 15 \\ κ - 2 = 4 \end{cases} \Leftrightarrow \boldsymbol{λ = 15}, \quad \boldsymbol{κ = 6}$$